Radioactivity in the Marine Environment

Understanding the Basics of Radioactivity

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To understand how radionuclides can be used in the marine environment, we must first explore:

1. Why radioisotopes exist.
2. What is radiation and radioactive decay?
3. What are key equations used to describe the radioactive decay process?
Why do we care?

- **Medicine**
  - Diagnostic & Therapy
  - Cancer treatments

- **Industrial Applications**
  - Quality Control
  - Sterilization of food, insects, etc.
  - Energy

- **Marine Science Applications**
  - Age Dating
  - Proxies & Tracers
The Basics: What is an atom?

N = Neutrons, neutral charge • mass \( N \) = 1.008665 atomic mass unit (amu)
Z = Protons (Atomic Number), positive charge • mass \( Z \) = 1.007825 amu
e = electrons, negative charge • mass \( e \) = 5.485 x 10^{-4} amu
A = Atomic Mass = \( N + Z \)
So \(^{23}_{11}\)Na has an \( A = 23 \), \( Z = 11 \), and \( N = 12 \)

Isotopes have identical chemical properties but a different atomic mass.

While the number of protons is the same, the number of neutrons in the nucleus differs.

http://www.nuclear-power.net/nuclear-power/reactor-physics/atomic-nuclear-physics/
Each pie chart shows the relative abundance of naturally occurring isotopes (both stable and long-lived unstable) of each element.
Some lightweight isotopes are unstable or radioactive but, all elements that have an atomic number (A) > 83 are radioactive.

Radioactivity – spontaneous change in the structure of the nucleus resulting in the transformation of the nucleus and the emission of particles (radiation) from the nucleus. This results in a loss of energy that changes the nucleus to a more stable configuration.
There are different kinds of radiation

Radiation is energy in the form of high-speed particles (or electromagnetic waves or photons). It can be ionizing or non-ionizing.

As you move from left to right, the wavelength (the distance between each peak or trough) decreases, and frequency increases.
There are many different types of non-ionizing radiation, but all lack the energy to alter atoms (e.g., visible light and microwaves).

*Ionizing radiation has enough energy to ionize atoms and can therefore change normal cellular functioning.*

Ionizing radiation is categorized by its strength or energy level and includes particles that are emitted from an unstable or radioactive nuclide.
The Discovery of Radioactivity: 
Best Failure Ever

Henri Becquerel in 1896.

Dr. Becquerel exposed potassium uranyl sulfate to sunlight and then placed it on photographic plates wrapped in black paper. He hypothesized that the uranium absorbed the sun’s energy and then emitted it in the form of x-rays.

This hypothesis was disproved on the 26th-27th of February, when his experiment "failed" because it was overcast in Paris. Dr. Becquerel decided to develop his photographic plates anyway. To his surprise, there were images that proved that the uranium emitted radiation without an external source of energy such as the sun. Becquerel had discovered radioactivity.

Won the Nobel Prize in Physics in 1903

Photographic plate from experiment
Where did all of these elements come from?  
And why do radioactive elements exist in nature?

The elements highlighted in red have radioactive isotopes that are most commonly used in Aquatic Science.
For our purposes, a galaxy is just a large collection of gas which is gravitationally bound.

This gas eventually clumps to make stars.
Elements are formed in two ways: **Fusion** and **Neutron Capture**

All stars derive their energy *through the thermonuclear fusion of light elements into heavy elements.*

**FUSION:** protons, neutrons and small elements crashing together to make bigger elements (very energetic process).

This cycle yields about $\sim 25$ MeV ($9.6 \times 10^{-13}$ calories) of energy...

http://butane.chem.uiuc.edu/pleshapley/GenChem1/L1/3.html
Summary of Fusion Reactions in Stellar interiors

Hydrogen Burning (> 3,000,000 K)
1) $\text{P} + \text{P} \rightarrow \text{^2H} + \text{positron} + \text{Energy}$
2) $\text{^2H} + \text{P} \rightarrow \text{^3He} + \text{Energy}$
3) $\text{^3He} + \text{^3H} \rightarrow \text{^4He} + \text{P} + \text{P} + \text{Energy}$

Carbon Nitrogen Cycle (> 10,000,000 K)
1) $\text{^{12}C} + \text{P} \rightarrow \text{^{13}N} + \text{Energy} \; (1.95 \text{ MeV})$
2) $\text{^{13}N} \rightarrow \text{^{13}C} + \text{positron} + \text{Energy}$
3) $\text{^{13}C} + \text{P} \rightarrow \text{^{14}N} + \text{Energy}$
4) $\text{^{14}N} + \text{P} \rightarrow \text{^{15}O} + \text{Energy} \; (7.35 \text{ MeV})$
5) $\text{^{15}O} \rightarrow \text{^{15}N} + \text{positron} + \text{Energy}$
6) $\text{^{15}N} + \text{P} \rightarrow \text{^{12}C} + \text{^4He} + \text{Energy} \; (4.96 \text{ MeV})$

Here $\text{P} = \text{Proton}$

Note that during this process, some of the neutrons that are added (creating an isotope of the same element), convert to a proton, thus changing the element!
Summary of Fusion Reactions in Stellar interiors

**Oxygen Burning (> 2,000,000,000 K)**

1. $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S} + \text{Energy}$
2. $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{P} + \text{P} + \text{Energy} (7.678 \text{ MeV})$
3. $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{S} + ^{4}\text{He} + \text{Energy} (1.500 \text{ MeV})$
4. $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{28}\text{Si} + ^{4}\text{He} + \text{Energy} (9.594 \text{ MeV})$

**Silicon Burning (> 3,000,000,000 K)**

1. $^{28}\text{Si} + ^{28}\text{Si} \rightarrow 7(^{4}\text{He}) + \text{Energy}$
2. $^{28}\text{Si} + 7(^{4}\text{He}) \rightarrow ^{56}\text{Ni} + \text{Energy}$
3. $^{28}\text{Si} + ^{28}\text{Si} \rightarrow ^{56}\text{Ni} + \text{Energy}$
4. $^{56}\text{Ni} \rightarrow ^{56}\text{Co} + \text{positron} + \text{Energy}$
5. $^{56}\text{Co} \rightarrow ^{56}\text{Fe} + \text{positron} + \text{Energy}$
After Fe, fusion becomes increasingly difficult...

**Neutron Capture** (less energetic)

Two processes:

**R-process --> Rapid** = capture of a neutron before a neutron-to-proton decay can occur (neutron $t_{1/2} = 12 - 15$ minutes!!)

**S-process --> Slow** = Neutron capture --> decays into proton before another neutron is captured

**R-process**

1) $^{56}\text{Fe} + \text{N} \rightarrow ^{57}\text{Fe} + \text{Energy}$

2) $^{57}\text{Fe} + \text{N} \rightarrow ^{58}\text{Fe} + \text{Energy}$

3) $^{58}\text{Fe} + \text{N} \rightarrow ^{59}\text{Fe} + \text{Energy}$

**S-process**

$^{59}\text{Fe} \rightarrow ^{59}\text{Co} + \text{electron} + \text{Energy}$

**R-process**

$^{59}\text{Fe} + \text{N} \rightarrow ^{60}\text{Fe} + \text{N} \rightarrow ^{61}\text{Fe}$

$^{61}\text{Fe} \rightarrow ^{61}\text{Co} + \text{electron} + \text{Energy}$
The Origin of the Solar System Elements

- **Big Bang Fusion**
  - Hydrogen (H)
  - Lithium (Li)
  - Beryllium (Be)
  - Boron (B)

- **Cosmic Ray Fission**
  - Carbon (C)
  - Nitrogen (N)
  - Oxygen (O)

- **Merging Neutron Stars**
  - Sodium (Na)
  - Magnesium (Mg)

- **Exploding Massive Stars**
  - Boron (B)

- **Dying Low Mass Stars**
  - Silicon (Si)

- **Exploding White Dwarfs**
  - Iron (Fe)

Graphic created by Jennifer Johnson

Astronomical Image Credits: ESA/NASA/AASNova

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doi:10.1002/loe2.10010
What are the characteristics of a nucleus that determine stability?

The Liquid Drop Model

1) Spin Pairing (+)

Neutrons and protons are fermions. They have a spin (± ½) and they like to form pairs (Pauli Exclusion Principle).

<table>
<thead>
<tr>
<th>A (n+z)</th>
<th>Z (number of protons)</th>
<th>N (number of neutrons)</th>
<th>Number of Stable Isotopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>Even</td>
<td>Even</td>
<td>156</td>
</tr>
<tr>
<td>Odd</td>
<td>Even</td>
<td>Odd</td>
<td>50</td>
</tr>
<tr>
<td>Odd</td>
<td>Odd</td>
<td>Even</td>
<td>48</td>
</tr>
<tr>
<td>Even</td>
<td>Odd</td>
<td>Odd</td>
<td>5</td>
</tr>
</tbody>
</table>

H, Li, B, N, Ta
What are the characteristics of a nucleus that determine stability?

2) Shell Binding (+)

Orbitals (electronic and nuclear) like to be filled!
“Magic Numbers” are when those orbitals are completely filled.

2 (i.e., an S orbital), 8 (S+P), 20 (S+P+D), 28, 50, 82, 126

For example:

19K has 3 stable isotopes
20Ca has 6 stable isotopes  MAGIC!
21Sc has 1 stable isotope

3) Surface (or Volume) Tension (-)

Surface tension is related to size. The higher the number of neutrons and protons, the lower the surface tension.
What are the characteristics of a nucleus that determine stability?

4) **Coulomb Repulsion (-)**

Like charges repel, meaning that a nucleus with more protons has more internal repulsion. As a result, it is easier to add neutrons (no charge) versus protons which are + charged!
What are the characteristics of a nucleus that determine stability?

Binding Energy reflects these four processes and is the energy that would be required to disassemble the nucleus of an atom into its component parts (protons, neutrons, etc., or even smaller atoms).

Ever notice that when you add the number of protons and neutrons together in an atom and compare it to what it actually weighs, you get more?

This is called the mass defect \( \Delta M \)

Example: \( ^{23}_{11}\text{Na} = (11 \times 1.007825) + (12 \times 1.008665) = 23.19006 \text{ amu} \)

Actual \( ^{23}_{11}\text{Na} = 22.98977 \text{ amu} \) \( \Delta M = 0.20236 \text{ amu} \)
Curve of the Binding Energy per nucleon

- Shell Binding Spin Pairing
- Coulomb repulsion
- Surface Tension

- Fusion
- Note change of scale

Mass number

Mass number:
- $^4_2\text{He}$
- $^{12}_6\text{C}$
- $^{16}_8\text{O}$
- $^{20}_{10}\text{Ne}$
- $^{56}_{26}\text{Fe}$

Binding energy per nucleon, MeV:
- $A<20$
- $Fission$
Element Abundance therefore depends on a mixture of Binding Energy and formation mechanisms.

- Abundances of first 50 elements decrease exponentially with atomic number
- Abundances of the heavier elements are independent of atomic number
- Note anomalously high abundance of Fe
So now that we know how all elements are formed and why they exist in nature, let’s examine the specific decay mechanisms that remove the excess energy of radioactive elements and enables them to become more stable.

Nuclei will transform in such a way to increase their Binding Energy per nucleon!

Alpha Decay

Emission of a helium nucleus, which contains two protons and two neutrons (but no electrons).

\[ ^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + \gamma \]

The \( \alpha \)-particle takes most (but not all) of the decay energy (it is the lightest).

Alpha decay occurs predominantly with \( A > 82 \) and its energy is specific to the radionuclide.
Conversion of a neutron into a proton and a beta particle escapes (a high-energy electron) from the nucleus. Note that the mass number does not change and there is a negligible effect on atomic weight.
Beta particles are emitted with a range of energies.

Remember the Conservation of Momentum Law?
Now there are **three** particles to deal with:

\[ zP_{\text{arent}} \rightarrow z+1D_{\text{aughter}} + \beta + \nu \]

This results in an **infinite** number of ways to share momentum.

Where \( E_{\text{MAX}} \) is the characteristic maximum Beta emission energy of a specific radionuculide.
Conversion of nuclear energy to electromagnetic energy (and the loss of energy [photon] from a neutron), note that the atomic number does not change.

Nearly always occurs with alpha and beta emissions and energy is specific to the radionuclide.
Alpha (α) particles: Most densely ionizing, but least penetrating. This means that cells can be protected or shielded from damage by alpha particles by clothing. Even the dead outer layer of your skin will protect you from damage from alpha particles. However, if alpha emitters are inhaled or ingested or get into a cut on the skin, they can cause damage to cells. When alpha particles are emitted inside the body, the surrounding cells are damaged.

Beta (β) particles: Usually less energetic, but more penetrating. Can travel several feet through air, but are stopped with denser materials such as wood, glass or aluminum foil depending on their energy. They can travel a few millimeters inside tissue.

Gamma (γ) rays: High-energy electromagnetic energy waves and the most penetrating type of radiation. Cells must be shielded from gamma rays with concrete, lead or steel. Not all may do cellular damage, but they must interact with the material to do so.
There are *many different types of particles* that are emitted from the nucleus during radioactive decay.

<table>
<thead>
<tr>
<th>Mode of decay</th>
<th>Participating particles</th>
<th>Daughter nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decays with emission of nucleons:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha decay</td>
<td>An alpha particle ((A = 4, Z = 2)) emitted from nucleus</td>
<td>((A - 4, Z - 2))</td>
</tr>
<tr>
<td>Proton emission</td>
<td>A proton ejected from nucleus</td>
<td>((A - 1, Z - 1))</td>
</tr>
<tr>
<td>Neutron emission</td>
<td>A neutron ejected from nucleus</td>
<td>((A - 1, Z))</td>
</tr>
<tr>
<td>Double proton emission</td>
<td>Two protons ejected from nucleus simultaneously</td>
<td>((A - 2, Z - 2))</td>
</tr>
<tr>
<td>Spontaneous fission</td>
<td>Nucleus disintegrates into two or more smaller nuclei and other particles</td>
<td></td>
</tr>
<tr>
<td>Cluster decay</td>
<td>Nucleus emits a specific type of smaller nucleus ((A_1, Z_1)) smaller than, or larger than, an alpha particle</td>
<td>((A - A_1, Z - Z_1) + (A_1, Z_1))</td>
</tr>
<tr>
<td>Different modes of beta decay:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta^-) decay</td>
<td>A nucleus emits an electron and an electron antineutrino</td>
<td>((A, Z + 1))</td>
</tr>
<tr>
<td>Positron emission ((\beta^+) decay)</td>
<td>A nucleus emits a positron and an electron neutrino</td>
<td>((A, Z - 1))</td>
</tr>
<tr>
<td>Electron capture</td>
<td>A nucleus captures an orbiting electron and emits a neutrino; the daughter nucleus is left in an excited unstable state</td>
<td>((A, Z - 1))</td>
</tr>
<tr>
<td>Bound state beta decay</td>
<td>A nucleus beta decays to electron and antineutrino, but the electron is not emitted, as it is captured into an empty K-shell; the daughter nucleus is left in an excited and unstable state. This process is suppressed except in ionized atoms that have K-shell vacancies.</td>
<td>((A, Z + 1))</td>
</tr>
<tr>
<td>Double beta decay</td>
<td>A nucleus emits two electrons and two antineutrinos</td>
<td>((A, Z + 2))</td>
</tr>
<tr>
<td>Double electron capture</td>
<td>A nucleus absorbs two orbital electrons and emits two neutrinos – the daughter nucleus is left in an excited and unstable state</td>
<td>((A, Z - 2))</td>
</tr>
<tr>
<td>Electron capture with positron emission</td>
<td>A nucleus absorbs one orbital electron, emits one positron and two neutrinos</td>
<td>((A, Z - 2))</td>
</tr>
<tr>
<td>Double positron emission</td>
<td>A nucleus emits two positrons and two neutrinos</td>
<td>((A, Z - 2))</td>
</tr>
<tr>
<td>Transitions between states of the same nucleus:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isomeric transition</td>
<td>Excited nucleus releases a high-energy photon (gamma ray)</td>
<td>((A, Z))</td>
</tr>
<tr>
<td>Internal conversion</td>
<td>Excited nucleus transfers energy to an orbital electron, which is subsequently ejected from the atom</td>
<td>((A, Z))</td>
</tr>
</tbody>
</table>
A single radioisotope can decay by many pathways... but they are **SET** pathways!

[Diagram showing decay pathways for potassium-40 (K) to argon-39 (Ar) and calcium-40 (Ca).]

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doi:10.1002/loe2.10010

http://www4vip.inl.gov/gammaray/catalogs/samples.shtml
**Fission**

Either a nuclear reaction or a *radioactive decay process* in which the nucleus of an atom splits into smaller parts (lighter nuclei). The fission process often produces free neutrons and photons and releases a very large amount of energy even by the energetic standards of radioactive decay.

http://physics.tutorvista.com/modern-physics/fission.html
U-Th series decay chains

Decay chart of the naturally occurring radionuclides $^{238}\text{U}$, $^{235}\text{U}$, and $^{232}\text{Th}$, and their radioactive daughters.

<table>
<thead>
<tr>
<th>Element</th>
<th>U-238 Series</th>
<th>Th-232 Series</th>
<th>U-235 Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium</td>
<td>U-238 4.5 x 10^9 y</td>
<td>Th-232 1.4 x 10^10 y</td>
<td>U-235 7.0 x 10^8 y</td>
</tr>
<tr>
<td>Protactinium</td>
<td>Pa-234 1.2 m</td>
<td>Ac-228 6.1 h</td>
<td>Pa-231 32800 y</td>
</tr>
<tr>
<td>Thorium</td>
<td>Th-234 24.1 d</td>
<td>Th-232 1.91 y</td>
<td>Th-231 25.5 h</td>
</tr>
<tr>
<td></td>
<td>Th-230 75400 y</td>
<td>Th-231 18.7 d</td>
<td></td>
</tr>
<tr>
<td>Actinium</td>
<td></td>
<td>Ac-227 21.8 y</td>
<td></td>
</tr>
<tr>
<td>Radium</td>
<td>Ra-226 1600 y</td>
<td>Ra-228 5.75 y</td>
<td>Ra-223 11.4 d</td>
</tr>
<tr>
<td>Francium</td>
<td></td>
<td>Ra-224 3.7 d</td>
<td></td>
</tr>
<tr>
<td>Radon</td>
<td>Rn-222 3.8 d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Astatine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polonium</td>
<td>Po-218 3.1 m</td>
<td>Bi-214 19.9 m</td>
<td></td>
</tr>
<tr>
<td>Bismuth</td>
<td>Pb-214 26.8 m</td>
<td>Pb-210 5.0 d</td>
<td></td>
</tr>
<tr>
<td>Lead</td>
<td>Pb-210 22.3 y</td>
<td>Pb-206 stable</td>
<td>Pb-207 stable</td>
</tr>
</tbody>
</table>

α Decay: Z: -2, N: -4
β Decay: Z: +1, N: ±0
Decay Series of Short-lived radionuclides

<table>
<thead>
<tr>
<th>Element</th>
<th>Symbol</th>
<th>Mass Number</th>
<th>Particle Reactivity in Seawater</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-238</td>
<td></td>
<td>4.5 x 10^9 y</td>
<td>Low</td>
</tr>
</tbody>
</table>

Figure re-drawn from: Rutgers van der Loeff (2014)
Radioactive Decay: How does the process actually work?

Radioactive decay is a **game of chance**. One cannot pick out a single nucleus and predict how long it will be until it undergoes radioactive decay. However, each unstable nucleus has a specific probability of decaying in a given time interval. **In sufficient numbers**, the probability of decay becomes well defined.

The Poisson distribution looks at the period of time for a fixed number of events. The Poisson distribution formula thus provides a probability of decay when the average decay rate is known.

http://www.icse.xyz/discuss/poisson/graphs.html
Radioactive Decay: How does the process actually work?

Every element decays with a specific half-life!

$$t_{1/2} = \frac{0.693}{\lambda}$$

Where $\lambda = \text{Radioactive Decay Constant}$
Radionuclides act as **CLOCKS** or **RATE TRACERS** for a variety of processes.
Radioactive decay is a rate function.

\[
\frac{dN}{dt} = -\lambda N = A = \text{Activity}
\]

\(N\) = number of atoms
\(\lambda\) = radioactive decay constant

\(\lambda = \frac{0.693}{t_{\frac{1}{2}}}\)

\(t_{\frac{1}{2}}\) = time it takes for half of the initial number of atoms to decay away

\(T = \frac{1}{\lambda} = \text{mean life of a radionuclide}\)
Radioactive decay is a rate function.

\[
\frac{dN}{dt} = -\lambda N = A = \text{Activity}
\]

\[
\frac{dN}{N} = -\lambda dt
\]

\[
\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda dt
\]

\[
\ln N - \ln N_0 = -\lambda (t - 0)
\]

\[
\ln \left( \frac{N}{N_0} \right) = -\lambda t
\]

\[
e^{\ln\left( \frac{N}{N_0} \right)} = e^{-\lambda t}
\]

\[
\frac{N}{N_0} = e^{-\lambda t}
\]

\[
N = N_0 e^{-\lambda t}
\]

\(N = \text{number of atoms}\)

\(\lambda = \text{radioactive decay constant}\)

\(N_0 = \text{number of atoms at time } t = 0\)

\(\lambda = 0.693/t_{1/2}\)

\(t_{1/2} = \text{time it takes for half of the initial number of atoms to decay away}\)

\(T = 1/\lambda = \text{mean life of a radionuclide}\)

The basic equation for radioactive decay!!
By comparing the number of parent and daughter atoms in a sample, we can estimate the amount of time since the sample was created. In the animation, the radioactive isotopes are represented by red circles, the decay products are the blue circles and the neutral isotopes are the green circles. Note that this example is for decay to a stable daughter.
The “classic” radioactive decay equation

**Case of a stable daughter (Geochronology in a nutshell)**

Because each parent \((N_p)\) atom that is lost to decay produces a daughter atom, we should be able to determine the number of parent atoms at \(t = 0\) \((N_p^0)\) by summing the number of parent atoms present today and the number of daughter atoms \((N_d^*)\) produced by decay of the parent since \(t = 0\) (assuming a closed system).

\[
N_p = N_p^0 e^{-\lambda t}
\]

\[
N_p^0 = N_p + N_d^*
\]

\[
N_p^0 = \frac{N_p^t}{e^{-\lambda t}} = N_p e^{\lambda t} = N_p + N_d^*
\]

\[
N_d^* = N_p e^{\lambda t} - N_p = N_p (e^{\lambda t} - 1)
\]
The “classic” radioactive decay equation

Calculate the time ($t$) elapsed since the composition of the sample was fixed to solve the equation for $t$.

\[ t = \frac{1}{\lambda} \ln \left[ 1 + \frac{N_d^*}{N_p} \right] \]

The total number of daughter atoms equals those present initially plus those produced by decay of the parent since the composition of the sample was fixed. What if you have other daughter atoms?

\[ N_d = N_d^0 + N_d^* + N_d \]

\[ N_d = N_d^0 + N_p \left( e^{\lambda t} - 1 \right) \]

Measure $N_d$ and $N_p$, but **ESTIMATE** $N_d^0$.

In most cases: This is the big unknown!!
Let’s get more complicated

The Case of the Radioactive Daughter

A \[\rightarrow\] B \[\rightarrow\] C \[\rightarrow\] ..........

\[N_1, \lambda_1\] \[N_2, \lambda_2\] \[N_3, \lambda_3\]

Daughter B forms at the rate of parent A decay, but B also decays. How do we find the activity of B at a particular time (The Bateman Equations)?

\[\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2\]

\[N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 \left[ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right] + N_2^0 e^{-\lambda_2 t}\]

Bateman (1910)
The case of the Radioactive Daughter

4 Cases

1. \( N_1 t_{1/2} < N_2 t_{1/2} \)
   - The half-life of the parent is shorter than that of the daughter.

2. \( N_1 t_{1/2} \sim N_2 t_{1/2} \)
   - The half-lives of the parent and daughter are similar.

3. \( N_1 t_{1/2} > N_2 t_{1/2} \)
   - The half-life of the parent is longer than that of the daughter.

4. \( N_1 t_{1/2} \gg N_2 t_{1/2} \)
   - The half-life of the parent is much longer than that of the daughter.
Case 1. $N_1 t_{1/2} < N_2 t_{1/2}$

The half-life of the parent is shorter than that of the daughter.

The graph shows the decay of different isotopes over time. The isotopes include:
- 218Po (3.1 m)
- 214Pb (26.9 m)
- 214Bi (19 m)
- 214Po (164 us)
- 210Pb (22.3 y)

The decay processes are indicated by α and β symbols.
The case of the Radioactive Daughter

**Case 2.** $N_1 t_{1/2} \sim N_2 t_{1/2}$

The half-lives of the parent and daughter are similar.

Graph showing decay curves for different isotopes: 135I (6.9 h), 135Xe (9.2 h), and 135Cs (2.3 My).
The case of the Radioactive Daughter

**Case 3.** $N_1 t_{1/2} > N_2 t_{1/2}$  
The half-life of the parent is longer than that of the daughter.

Note that when $t > \ln(\lambda_2/\lambda_1)/(\lambda_2-\lambda_1)$, the activity ratio will approach: $\lambda_2/(\lambda_2-\lambda_1)$. 

Claudia Benitez-Nelson  
doi:10.1002/loe2.10010
Case 4. \( N_1 t_{1/2} \gg N_2 t_{1/2} \)

The half-life of the parent is much longer than that of the daughter.

![Graph showing the decay of 238U and 234Th over time.]

Claudia Benitez-Nelson

DOI: 10.1002/loe2.10010
U-Th series decay chains

For the naturally occurring radionuclides: $^{238}\text{U}$, $^{235}\text{U}$, and $^{232}\text{Th}$, the half-lives of the parent nuclides are much much longer than their daughter products (Case 4!).

<table>
<thead>
<tr>
<th>Element</th>
<th>U-238 Series</th>
<th>Th-232 Series</th>
<th>U-235 Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium</td>
<td>U-238 4.5x10^9 y</td>
<td>U-234 245500 y</td>
<td>U-235 7.6x10^9 y</td>
</tr>
<tr>
<td>Protactinium</td>
<td>Pa-234 1.2 m</td>
<td>Th-232 1.4x10^10 y</td>
<td>Pa-231 32800 y</td>
</tr>
<tr>
<td>Thorium</td>
<td>Th-234 24.1 d</td>
<td>Th-230 75400 y</td>
<td>Th-228 1.91 y</td>
</tr>
<tr>
<td>Actinium</td>
<td></td>
<td>Ac-228 6.1 h</td>
<td>Th-231 25.5 h</td>
</tr>
<tr>
<td>Radium</td>
<td>Ra-226 1600 y</td>
<td>Ra-228 5.75 y</td>
<td>Ac-227 21.8 y</td>
</tr>
<tr>
<td>Francium</td>
<td></td>
<td>Ra-224 3.7 d</td>
<td></td>
</tr>
<tr>
<td>Radon</td>
<td>Rn-222 3.8 d</td>
<td></td>
<td>Ra-223 11.4 d</td>
</tr>
<tr>
<td>Astatine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polonium</td>
<td>Po-218 3.1 m</td>
<td>Po-214 0.00014 s</td>
<td>Po-210 138 d</td>
</tr>
<tr>
<td>Bismuth</td>
<td>Pb-214 26.8 m</td>
<td>Bi-214 19.9 m</td>
<td>Pb-208 stable</td>
</tr>
<tr>
<td>Lead</td>
<td>Pb-210 22.3 y</td>
<td>Bi-210 5.0 d</td>
<td>Pb-208 stable</td>
</tr>
</tbody>
</table>

α Decay  
\[ Z: - 2 \]  
\[ N: - 4 \]

β Decay  
\[ Z: + 1 \]  
\[ N: \pm 0 \]

Decay Series of Short-lived radionuclides

Element Symbol  
U-238  
4.5 x 10^9 y

Half life

Particle Reactivity in Seawater

- Low
- Medium
- High

Figure re-drawn from: Rutgers van der Loeff (2014)
Case 4. $N_1^{t_{1/2}} \gg N_2^{t_{1/2}}$

The half-life of the parent is much longer than that of the daughter.

\[
\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \rightarrow \quad N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 [e^{-\lambda_1 t} - e^{-\lambda_2 t}] + N_2^0 e^{-\lambda_2 t}
\]

When $t \gg \frac{1}{\lambda_2}$ and $\lambda_2 \gg \lambda_1$ \(\rightarrow\) \(\lambda_1 N_1 = \lambda_2 N_2\)

\[A_1 = A_2 \]

Secular Equilibrium

Activity takes into account the radioactive decay component (and is what we actually measure).
Units of Radioactivity

- **Curie (Ci)**
  Originally based on the disintegration rate of 1 g of radium, now defined as the quantity of any radioactive nuclide in which the number of disintegrations per second (dps) is $3.7 \times 10^{10}$.

- **Becquerel (Bq)**
  The SI unit, defined as the amount of ionizing radiation released when an element spontaneously emits energy as a result of the radioactive decay (or disintegration) of an unstable atom. 1 Bq = 1 disintegration per second (dps). 60 dps = 1 disintegration per minute (dpm).

- **Disintegrations per minute (dpm)**
  *A favorite unit for oceanographers*
Example: Consider the $^{232}\text{Th}$ decay to $^{224}\text{Ra}$.

Activity = $N\lambda$

$\lambda = 0.693/t_{1/2}$

Where $N =$ number of atoms, $\lambda =$ decay constant, and $t_{1/2} =$ the half-life

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>$^{232}\text{Th}$</th>
<th>$^{224}\text{Ra}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-life ($t_{1/2}$)</td>
<td>$1.4 \times 10^{10}$ y</td>
<td>3.7 d</td>
</tr>
<tr>
<td>Atoms kg$^{-1}$</td>
<td>$1.06 \times 10^{16}$</td>
<td>$7.60 \times 10^{3}$</td>
</tr>
<tr>
<td>Dpm kg$^{-1}$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Bq kg$^{-1}$</td>
<td>0.017</td>
<td>0.017</td>
</tr>
</tbody>
</table>
The case of the Radioactive Daughter

**Case 4.** $N_1 t_{1/2} \gg N_2 t_{1/2}$

The half-life of the parent is much longer than that of the daughter.
The plot of parent and daughter activity in secular equilibrium is:

\[ A_{N2}(t) \approx A_{N1} \left( 1 - \exp\left(-\lambda_{N2}t\right) \right) \]

Time until daughter reaches secular equilibrium is \( \approx \frac{4.6}{\lambda_D} \)
In the absence of external forces, the activity of the parent and daughter should be equal. The key is to find instances when secular equilibrium has been disrupted in a Case 4 situation by processes that cause fractionation between parent and daughter owing to differences in their physico-chemical properties. Studies of these non-equilibrium systems may be used to determine rates of the disruptive process.
Summary

1) Radioisotopes are produced and decay according to mechanisms and pathways that are known and measurable.

2) There are key equations used to describe the radioactive decay process, and these equations can be vastly simplified under certain conditions. This allows radionuclides to be more easily used as clocks/rate tracers in a variety of processes in the environment.
Acknowledgements

This work would not have been possible without the generous contributions and thoughtful comments of Drs. Robert Anderson, Kirk Cochran, Peter Santschi, Michiel Rutgers van der Loeff, and Alan Shiller. We wish to thank two anonymous reviewers who provided constructive comments that improved the presentation. Lectures would not have been possible without the outstanding assistance of graphic designer Jason Emmett.

This work was supported by the Scientific Committee on Ocean Research, SCOR Working Group 146: *Radioactivity in the Ocean, 5 Decades Later (RiO5)* and the Woods Hole Oceanographic Institution’s *Center for Marine and Environmental Radioactivity*.
References and Reading List

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Published by John Wiley & Sons, Inc.

Claudia Benitez-Nelson, Ken Buesseler, Minhan Dai, Michio Aoyama, Núria Casacuberta, Sabine Charmasson, Andy Johnson, José Marcus Godoy, Vladimir Maderich, Pere Masqué, Willard Moore, Paul J. Morris, John N. Smith

Radioactivity in the Marine Environment:
Understanding the Basics of Radioactivity

doi:10.1002/loe2.10010

e-Lecture received June 2016; accepted October 2018

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